

Chapter 6: Approximate Cell Decomposition

ted, the associated connectivity graph, denoted by G_i , is searched for channel connecting q_{init} to q_{goal} .

simple first-cut planning algorithm is the following:

1. Compute a rectangloid decomposition \mathcal{P}_1 of Ω . Set i to 1.
2. Search the connectivity graph G_i associated with the decomposition \mathcal{P}_i ; for a channel connecting the initial cell containing q_{init} to the goal cell containing q_{goal} . If the outcome of the search is an E-channel, return success. If it is an M-channel, proceed to the next step. Otherwise, return failure.
3. Let Π_i be the M-channel generated at Step 2. Set \mathcal{P}_{i+1} to \mathcal{P}_i . For every MIXED cell κ in Π_i , compute a rectangloid decomposition \mathcal{P}^κ of κ and set \mathcal{P}_{i+1} to $[\mathcal{P}_{i+1} \setminus \{\kappa\}] \cup \mathcal{P}^\kappa$. Set i to $i + 1$. Go to Step 2.

he search of G_i at Step 2 can be guided by various heuristics. In

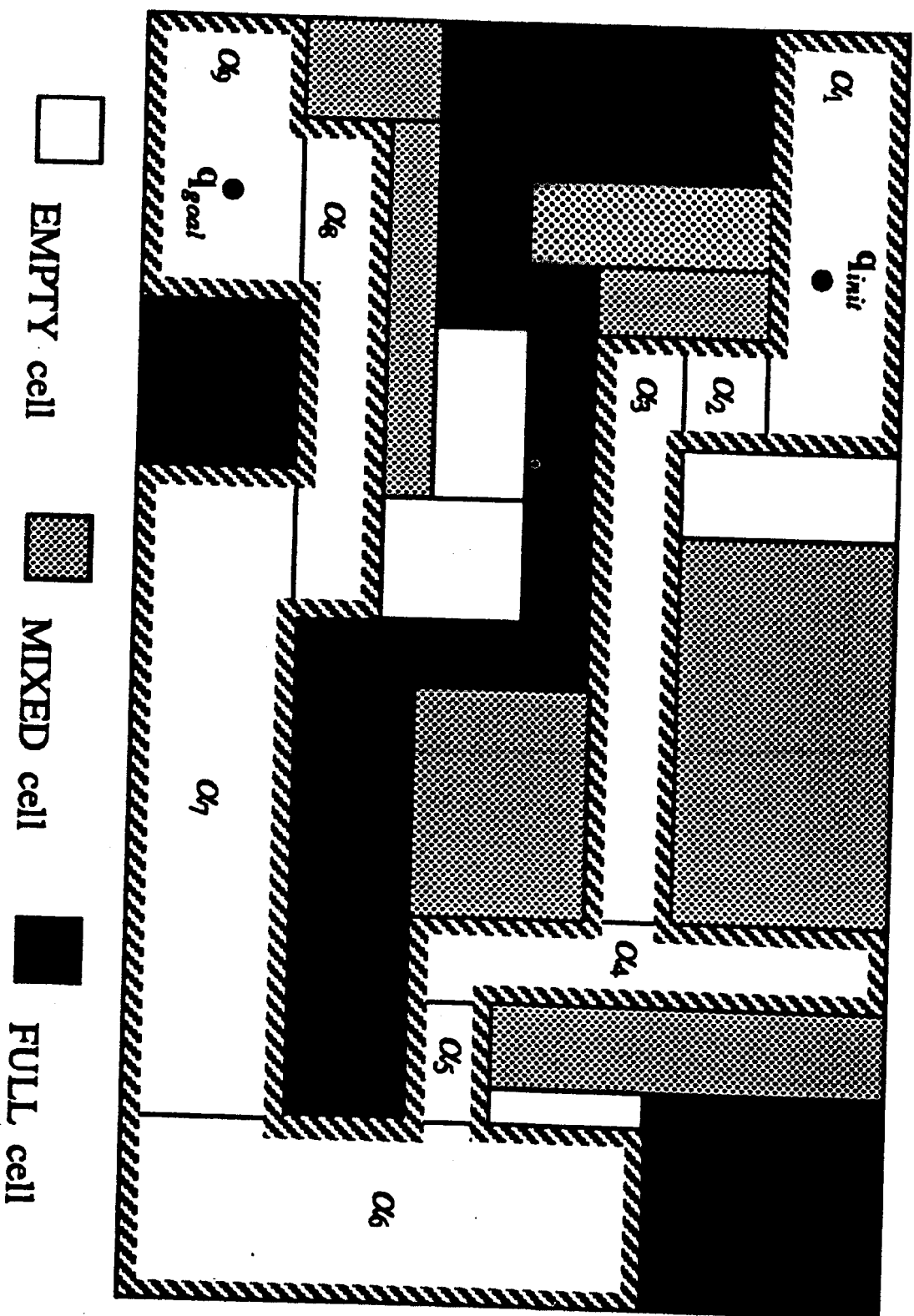
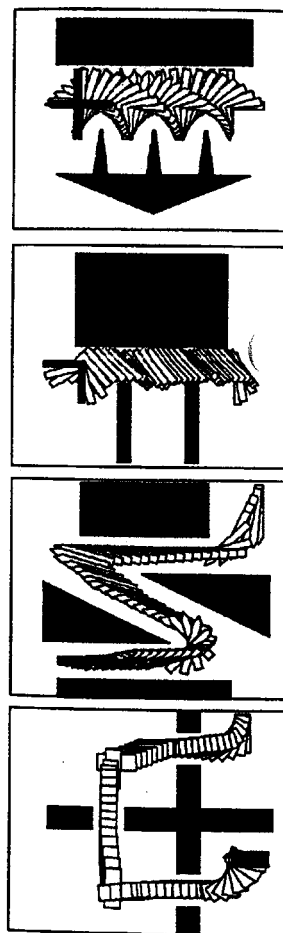


Figure 1. A channel is a sequence of adjacent cells which are either EMPTY or MIXED. If all the cells are EMPTY the channel is said to be an E-channel, otherwise it is said to be an M-channel. This figure shows an E-channel (striped contour) in a two-dimensional space. It connects the two cells that contain the

How to divide a mixed cell
for Hur and Label the
Subdivided cells?

"Divide + Label"



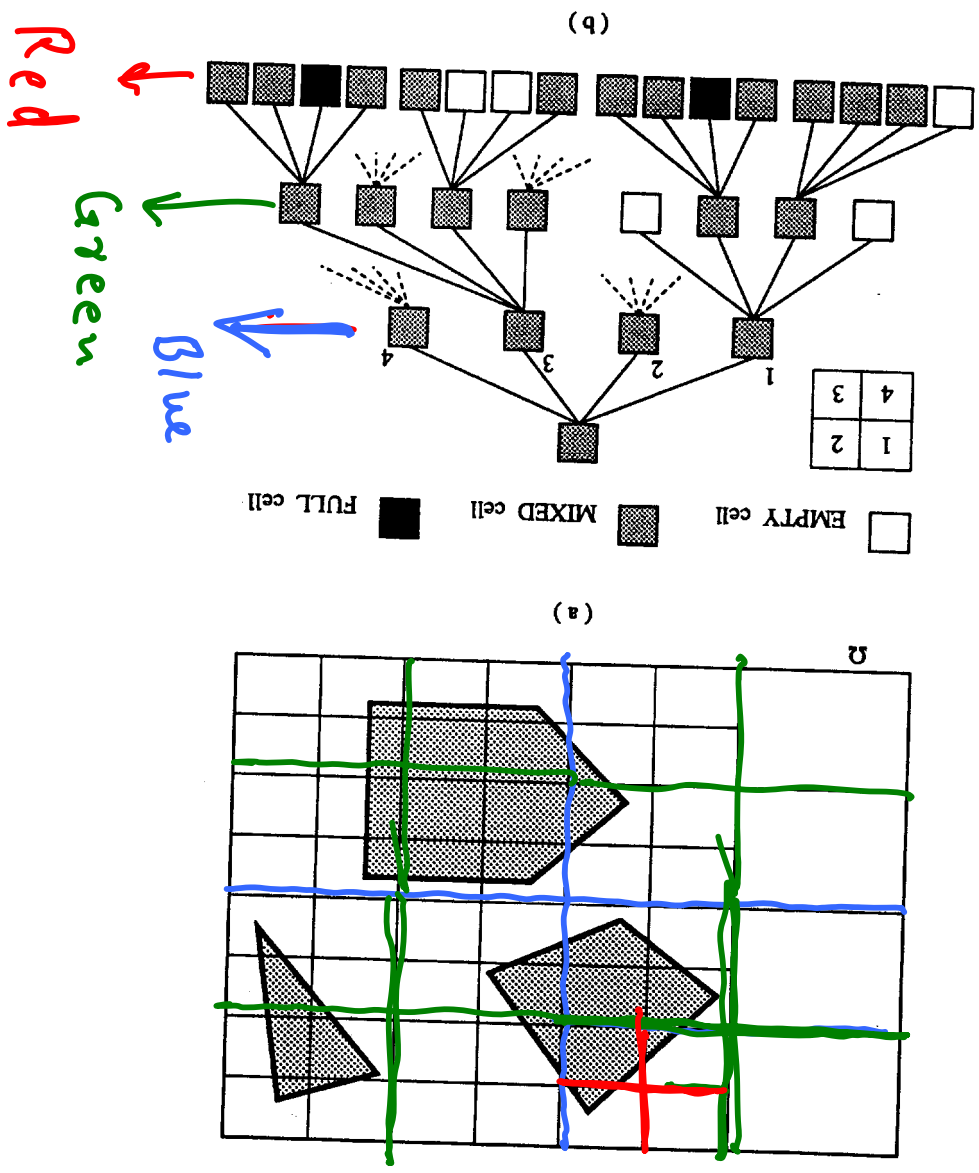
by a planner based on the
 us input problems [Zhu and
 polygon that can translate
 stacks.

Each edge of a cell k have the
 : each edge of k into two

If $m = 3$, it is called an
 of a two-dimensional

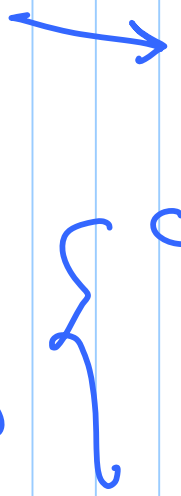
2 Divide-and-Label

Figure 4. A quadtree decomposition of Ω is obtained by recursively dividing Ω and the generated MIXED cells into smaller cells. The division of a cell creates four new rectangular cells of equal dimensions. Figure a shows the quadtree decomposition at depth 3 of a simple configuration space. Figure b shows a subset of the corresponding tree.



Cell Labeling:

Recall CB : $V \setminus X_{ij}$



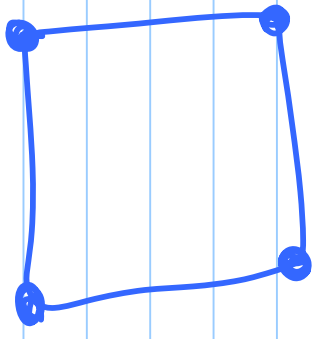
$i \in \#$ of convex

obs. obs.

first in \mathbb{R}^n : polyg. case $j \in \#$ of vertices

"Cells are convex" no rot.

So need to test only vertices

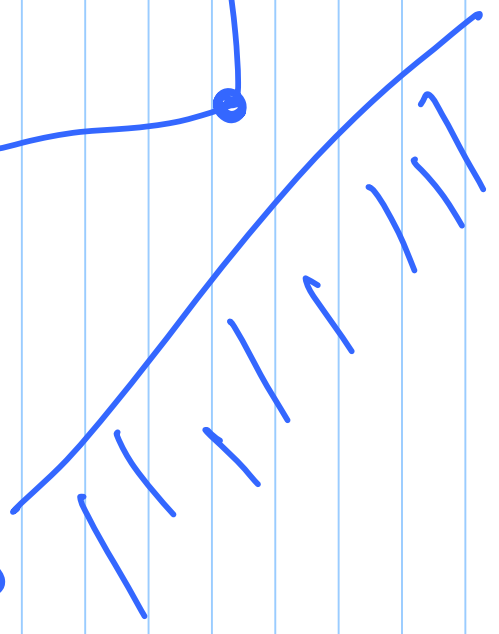


outside

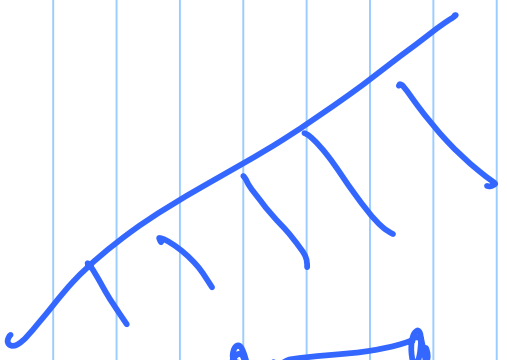
all vertices

do not

satisfy e_{ij}



e_{ij}



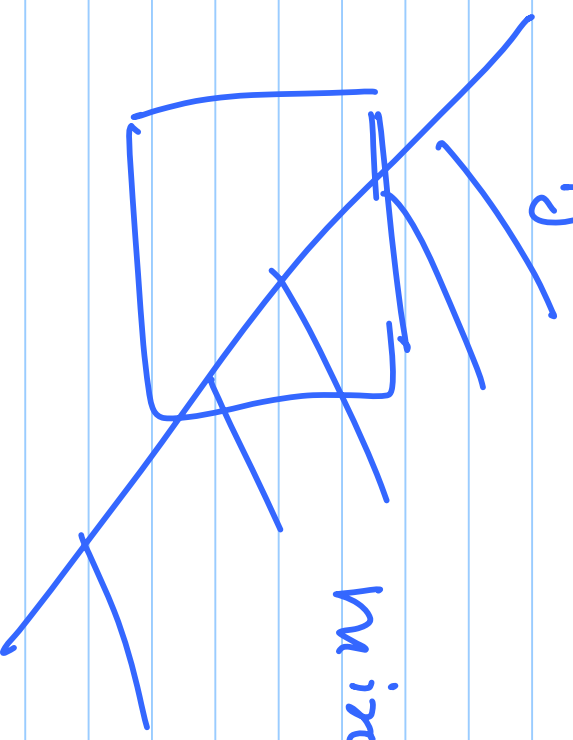
inside

all vertices

notify

e_{ij}

mixed



$$S_{\kappa'} = e_1 \vee (e_2 \wedge e_3)$$

$$S_{\kappa_1} = e_2 \wedge e_3$$

$$S_{\kappa_2} = e_2 \wedge e_3$$

$$S_{\kappa_3} = e_1$$

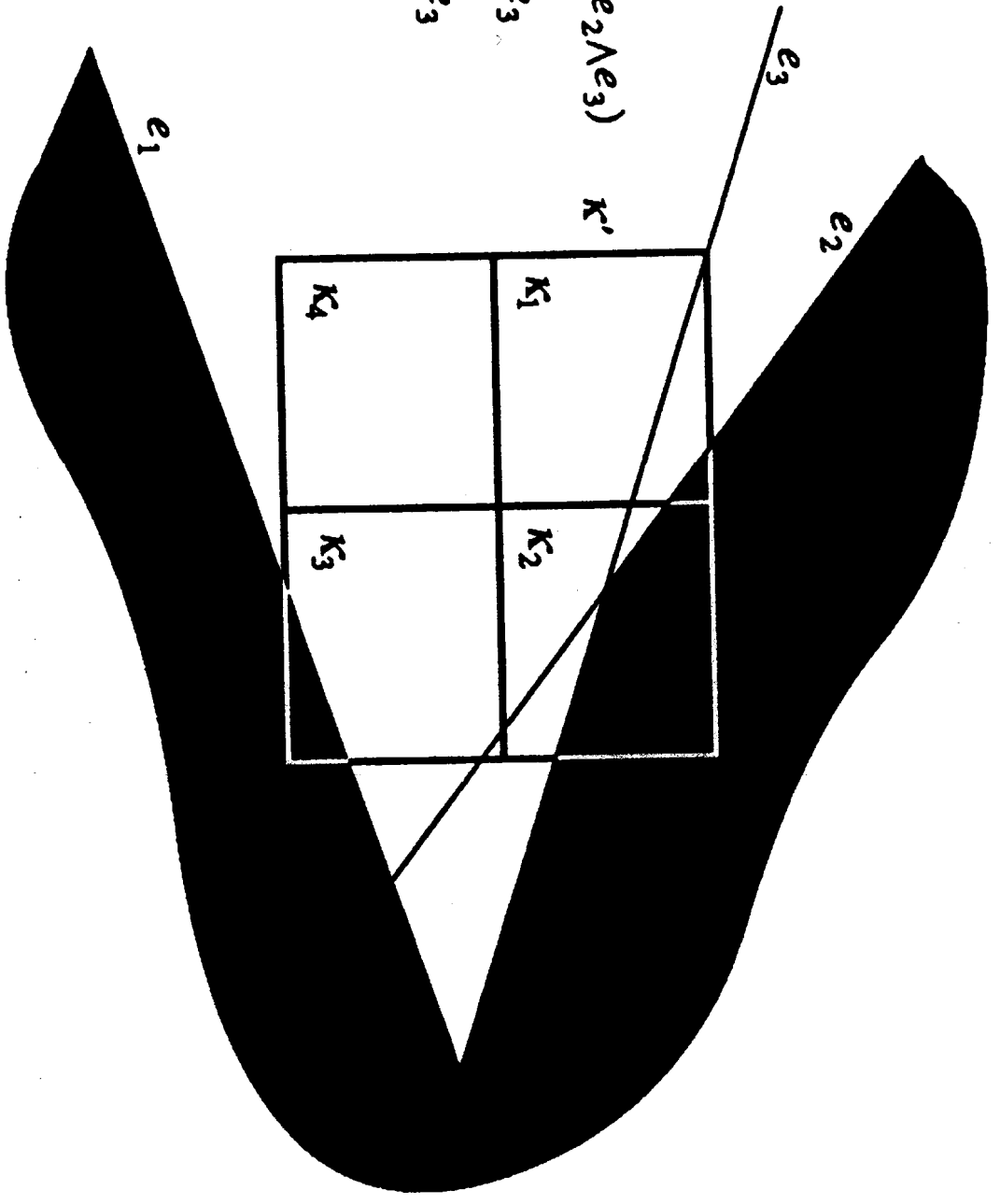


Figure 6. This figure illustrates the simplification of a C-sentence when new cells are created and labeled. The sentence $S_{\kappa'} = e_1 \vee (e_2 \wedge e_3)$ is associated with the MIXED cell κ' . When this cell is decomposed (in a quadtree fashion), four new cells denoted by κ_1 through κ_4 are generated. Both κ_1 and κ_2 are

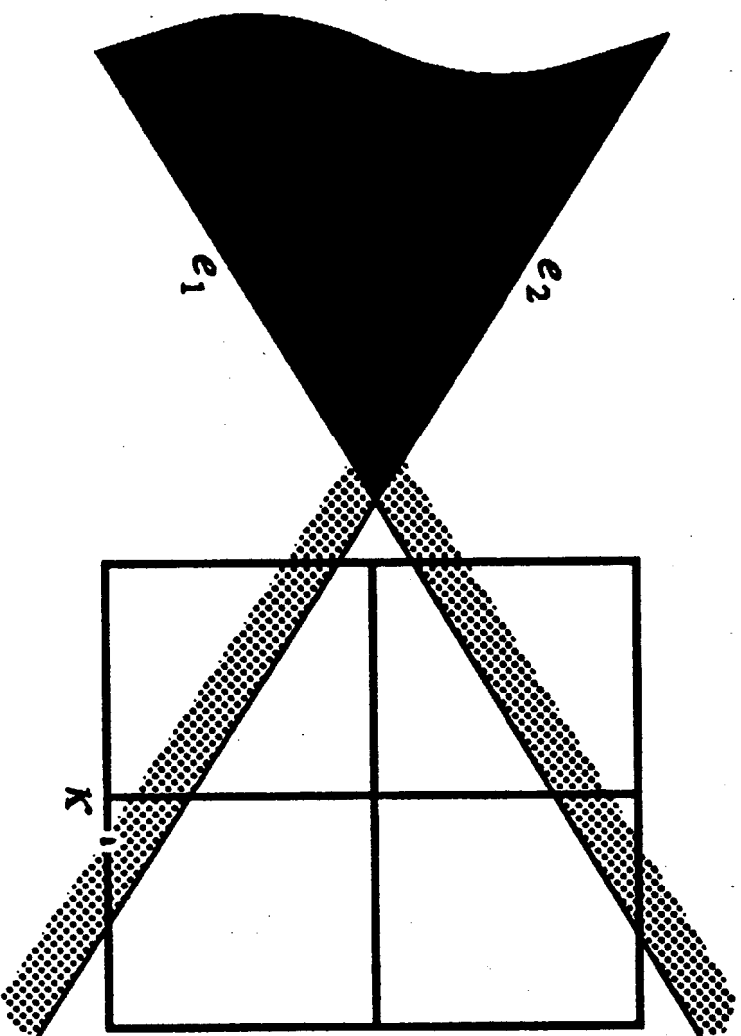
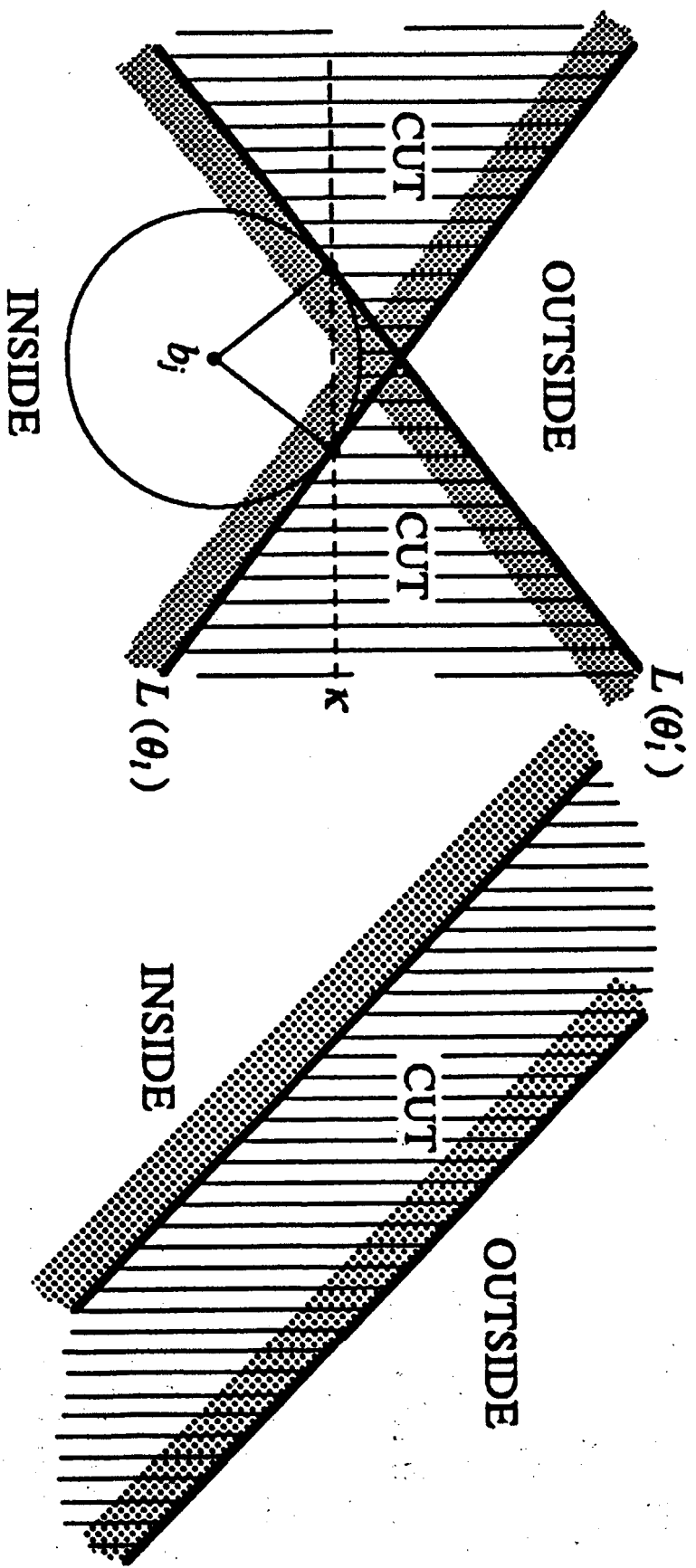


Figure 7. This figure illustrates how a cell κ gets labeled as MIXED though it has no intersection with the C-obstacle region. Assume that the C-sentence associated with the parent cell of κ is $e_1 \wedge e_2$. Since κ is cut by both e_1 and e_2 , κ is labeled as MIXED and the C-sentence $e_1 \wedge e_2$ is associated with it. However, no point in κ satisfies e_1 and e_2 *simultaneously*. This incorrect (but conservative) labeling results from the fact that each C-constraint is individually considered as a half-plane. The error is eventually corrected at a deeper level in the quadtree decomposition. (The “inside” side of a C-constraint line is shown

C : $\mathbb{R}^2 \times SO(1)$ or $\mathbb{R}^2 \times S^1$

polygon with trans + rot.



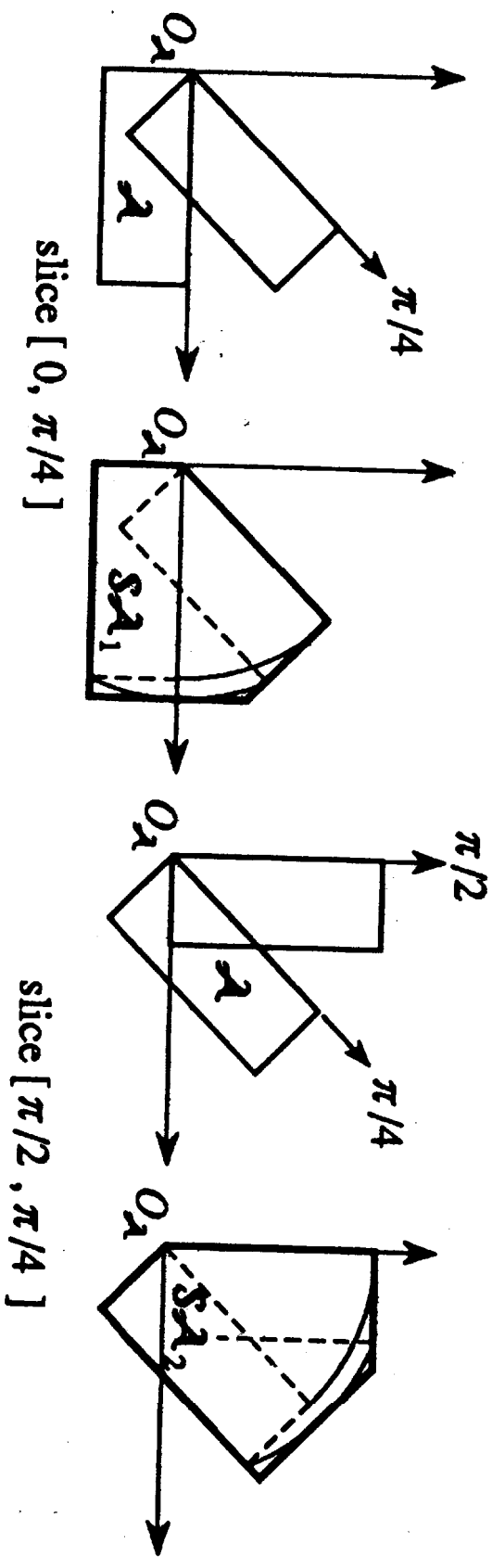
(a)

(b)

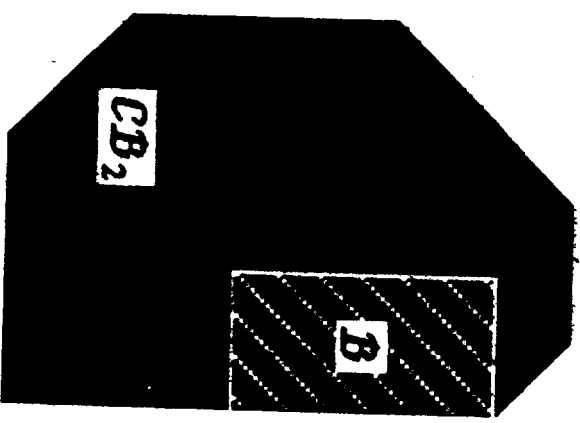
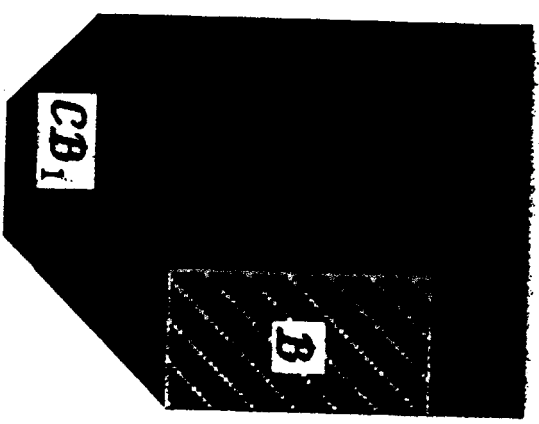
Figure 8. This figure illustrates the projection of the portion of the C-surface $a(\theta)x + b(\theta)y + c(\theta) = 0$ which is comprised in the angular interval $[\theta_i, \theta'_i]$ in the xy -plane. When the C-surface is of type A, the projection is the region swept out by a line rotating around an obstacle vertex b_j (Figure a). When swept out by a line rotating around an obstacle vertex b_j (Figure a). When the C-surface is of type B, the projection is the region swept out by a translate line parallel to an obstacle edge E_j^B (Figure b). In both cases, the project divides the plane into three regions designated by OUTSIDE, INSIDE, and

ORIENTATION SKILLS:

" Approximate " approach



(a)



(b)